

# Abstract PhDs in Logic III

## *Ambiguities for NF*

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Quine has laid the foundation of his new set theory in his article *New Foundations for Mathematical Logic* [1], naturally called New Foundations (NF). He wouldn't have imagined that we are still trying to prove the consistency or the inconsistency of his system for set theory. People have tried several ways to solve this problem, no one succeed, but a lot of interesting topics have emerged from these researches.

After giving a short introduction to NF and to the Theory of Types (TT), we will focus on a very elegant subject, say ambiguity, using the powerful link between NF and TT.

The very first notion of ambiguous cardinal (referred to as *typical ambiguity*) has been introduced within the framework of NF after the result by Specker in 1962 [3]. He showed that the existence of an *ambiguous model* of the TT (a model  $\langle M_0, M_1, M_2 \dots \rangle \models \text{TT}$  such that  $\langle M_0, M_1, \dots \rangle \equiv \langle M_1, M_2, \dots \rangle$ ) yields to the consistency of NF. We restrict our research to a particular case of ambiguous model: the natural structure over a non-empty set  $M$ :  $\langle \langle M \rangle \rangle = \langle M, \mathcal{P}(M), \dots \rangle$ . If  $\langle \langle M \rangle \rangle$  is ambiguous, we say that  $|M|$  is an ambiguous cardinal. The search of ambiguous cardinals has not been successful. The former idea was to study, as usually done when working with the problem of the consistency of NF, weaker consistent theories, like NFU, to enrich them to try to reach a consistent extension corresponding to NF. We have already proved how to find a model of NFU thanks to *weakly ambiguous cardinal* (a weaker notion of ambiguous cardinal), for instance. We clearly see that this method is finer than those usually written in the set theory ZFJ (ZF with an automorphism  $J$ ), typically based on the previous result of Specker in 1958 [2] on shifting models of TT.

We will then present another way to tackle the problem of the consistency of NF. First, we will study “stronger” theories, inconsistent with the existence of a certain kind of ambiguous cardinals, like ZFC, and by weakening them, we would like to lead to the inconsistency of NF. The point is to find interesting definitions of ambiguity in order to make it very fine. We are then working in ZF and some variant of it, making possible to internalize the definition of ambiguities. We will define ambiguity in a cumulative hierarchy over a set considered as a set of atoms. If time permits, we will present some other notions of ambiguities that are more or less powerful.

## References

- [1] Quine, W. V. O. , New foundations for mathematical logic, American Mathematical Monthly 44, pp.70-80, 1937.
- [2] Specker, E. P., Dualität, Dialectica 12, pp.451-465, 1958.
- [3] Specker, E. P., Typical ambiguity, *Logic, methodology and philosophy of science*, ed. E. Nagel, Stanford University Press, pp.116-123, 1962.