Relational semantics for linear logic

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Relational semantics, given by Kripke frames, play an essential role in the study of modal and intuitionistic logic [1]. They provide an intuitive interpretation of the logic and a means to obtain information about it. The possibility of applying semantical techniques to obtain information about a logic motivates the search for relational semantics in a more general setting.

Many logics are closely related to a certain class of algebraic structures. Kripke frames arise naturally from the duality theory for the algebraic structures associated to modal and intuitionistic logic. This insight has allowed a generalisation of Kripke frames to analogous semantics for a broader setting including substuctural logics [2].

In a joint project with Mai Gehrke and Lorijn van Rooijen we have developed relational semantics for linear logic. Linear logic was introduced by Jean-Yves Girard in 1987 [3]. Formulas in linear logic represent resources which may be used exactly once. Proof theoretically this is witnessed by the fact that contraction and weakening are not admissible in general. However, these structural rules are allowed in a controlled way by means of a new modality, the exponential !, which expresses the case of unlimited availability of a specific resource.

In [2] relational semantics for the fusion-implication fragment of linear logic are described. We will discuss our recent work in which we have used techniques from duality theory to derive the semantical structure corresponding to the linear negation and the exponential. Thereby we describe relational semantics for full linear logic. This work illustrates the strength of the use of duality theory in the search for relational semantics: it allows a modular and uniform treatment of additional operations and axioms.

Traditionally, so called phase spaces are used to describe semantics for linear logic [3]. These have the drawback that, contrary to our approach, they do not allow a modular treatment of additional axioms. However, the two approaches are related as will be explained in the talk.

References

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[3] J.-Y. Girard, Linear Logic, Theoretical Computer Science, vol. 50, p.1-102, 1987.