A Logic Approach to Pluralistic Ignorance

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In this paper, the possibility of modeling the phenomenon "pluralistic ignorance" from social psychology, using epistemic/doxastic logic, is investigated. It is shown that the straightforward formalization of pluralistic ignorance does not capture the phenomenon completely. What is needed in addition is an assumption about the interplay between different agents' beliefs or an assumption about how agents will change their beliefs about the entire group's beliefs based on information about a single agent's beliefs.

Pluralistic ignorance is a term from the social and behavioral sciences dating back to the work of Floyd H. Allport and Daniel Katz [5].[1] Citing Krech and Crutchfield ([6], pp. 388-89) pluralistic ignorance is a situation where "no one believes, but everyone thinks that everyone believes" [2]. Elaborated, pluralistic ignorance is the case in which a group of people share a false belief about the beliefs, norms, actions or thoughts of the other group members, and thus it is an important notion in understanding the social life. Examples of pluralistic ignorance are plenty in the social science literature. One example is the use of alcohol at (American) college campuses, where several studies have shown that many students feel much less comfortable drinking than they believe the average student to feel [8]. Another classical example is the class room example in which, after having presented the students with some difficult material, the teacher asks them whether they have any questions. Even though most students do not understand the material, there may fail to be any questions. Each student interpret the lack of questions from their fellow students as a sign that all the others understand the material and, in order to avoid being publicly displayed as the less intelligent student, they do not dare asking any questions themselves.

Misperceiving other people's norms or beliefs can often occur without it being a case of pluralistic ignorance. Pluralistic ignorance is the case of systematic errors in norm/belief estimation of others. Thus pluralistic ignorance is a genuine social phenomenon and not just some people holding wrong beliefs about other people's norms or beliefs. Another thing that characterizes most cases of pluralistic ignorance is how fragile it is. Just one public announcement of a private belief or norm will resolve the case of pluralistic ignorance. The two features, being a genuine social phenomenon and being resolvable by a public announcement, will play an important role in the attempt to model pluralistic ignorance using logic in this paper.

Pluralistic ignorance has been approached by formal methods before (for instance [2, 4]), but almost none of these methods have been logic. The only exception seems to be Vincent Hendrick's paper [4] where pluralistic ignorance is modeled using formal learning theory and logic. In this paper the tool will be classical modal logic in the form of doxastic/epistemic logic.

[1] See [7] for more on the coining of the Term "Pluralistic Ignorance".

Let $\mathbb{A}$ be a finite set of agents. Assuming pluralistic ignorance is a case where no agent believes φ, but every agent believe that everyone else believes φ, we can easily formalize it in doxastic logic as:

$$\bigwedge_{a\in\mathbb{A}}\Big(\neg B_a\varphi \wedge \bigwedge_{b\in\mathbf{A}\setminus\{a\}} B_aB_b\varphi\Big) \tag{1}$$

In standard doxastic logic as multi-modal **KD45** (see for instance [3]) such a formula is satis?able since a possible world model can easily be constructed satisfying the formula. However, due to the problem of combining multi-modal **KD45** and public announcement, we will use the plausibility models of Baltag and Smets [1] in which the formula (1) is also satisfiable.[2] In this framework, combining beliefs and public announcements is not problematic.

In the framework of Baltag and Smets, the classical notion of public announcement can be applied. I.e. a public announcement of φ will result in a new plausibility model where all the $\neg\varphi$-worlds have been removed. According to the second feature of pluralistic ignorance, pluralistic ignorance should be dissolved if just one agent announces his true beliefs. Still, a model satisfying (1) can be constructed so that, after an announcement of $!\neg B_b\varphi$, all agents still believe that all other agents than b believe φ. This will be explained into further detail in the full paper.

Thus, in the framework of Baltag and Smets, (1) does not capture pluralistic ignorance quite accurately, since it is not fragile to public announcement. However, being fragile to public announcement is something we can add by brute force to (1). Thus, here is a more accurate definition of pluralistic ignorance:

$$\bigwedge_{a\in\mathbb{A}}\Big(\neg B_a\varphi \wedge \bigwedge_{b\in\mathbf{A}\setminus\{a\}} B_aB_b\varphi\Big) \wedge \bigwedge_{a\in\mathbb{A}}\Big([!\neg B_a\varphi]\big(\bigwedge_{b,c\in\mathbb{A}} B_b\neg B_c\varphi\big)\Big)$$

Besides ensuring that the agents' beliefs are fragile, the second conjunct also reveals the importance of the second feature of pluralistic ignorance, namely that it is a social phenomenon. If it is announced that a does not believe φ then all the agents believe that no one else will believe φ. An essential point about pluralistic ignorance is exactly the fact that the agent's beliefs are not independent of each other. Thus, to model pluralistic ignorance logical models in which agents' beliefs are dependent on other agents' beliefs are needed. In many cases, the dependency between agents' beliefs is exactly what creates pluralistic ignorance in the first place.

Using public announcement, as in (2), is a dynamic way of capturing the dependency between agents' beliefs but there are other ways of capturing this. This is something that will be investigated further in the full paper as well. The full paper will focus on how epistemic aspects of different notions of pluralistic ignorance can be logically sound and how the pluralistic ignorance can be dissolved through announcements.

[2]In the framework of Baltag and Smets, there are several notions of beliefs; "plain" beliefs, safe beliefs, weak safe beliefs and strong beliefs. However, only for the notions of "plain" beliefs and strong beliefs is the formula (1) satisfiable. For safe beliefs and weak safe beliefs (1) is satisfiable since these notions of beliefs implies truth.

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